edexcel

Mark Scheme (Results)

January 2015

Pearson Edexcel International A Level
Core Mathematics 12
(WMA01_01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

January 2015
International A Level WMA01/01 Core Mathematics C12
Mark Scheme

Question Number	Scheme	Marks
1.	(a) x^2	B1 **[1]**
	(b) $\frac{1}{4}x^4$ or $\frac{1}{2^2}x^4$ or $0.25x^4$	B1, B1 **[2]**
		3 marks
	Notes	

(a) **B1**: This answer only

(b) **B1:** For $\frac{1}{4}x^k$ **as final answer**, k can even be 0. Also accept $\frac{1}{2^2}$ for B1 but 2^{-2} is not simplified and is B0

B1: for x to power 4 (independent mark) so kx^4 with k a constant (could even be 1) **as final answer**

n.b. Can score B0B1 or B1B0 or B0B0 or B1B1

Mark the final answer on this question

Also note : Candidates who misread question as $\sqrt{2x^3} \div \sqrt{\frac{32}{x^2}}$ should get $\frac{1}{4}x^{\frac{5}{2}}$ This is awarded B1B0

Special case: The answer $\left(\frac{1}{\sqrt{2}}x\right)^4$ is awarded B0 B1 as x may be in a bracket with power 4 outside.

Question Number	Scheme	Marks
2.	<table><tr><td>x</td><td>2</td><td>5</td><td>8</td><td>11</td></tr><tr><td>y</td><td>8.485</td><td>2.502</td><td>1.524</td><td>1.100</td></tr></table>	
(a)	State $h = 3$, or use of $\frac{1}{2}\times 3$	B1 aef
	$\underline{\{8.485 + 1.100 + 2(\ 2.502 + 1.524\ )\}}$ For structure of $\{\ldots\ldots\ldots\ldots\ldots\}$	M1A1
	$\frac{1}{2}\times 3 \times \underline{\{17.637\}}$ $(= 26.4555)$ $=$ awrt 26.46	A1
		[4]
(b)	Adds 9+……	M1
	half of **their answer from (a)** seen (allow use of half of 26.4555)	M1
	So required estimate = 9 + 13.23 = 22.23	A1
		[3]
		7 marks
(b)	Way 2: **Begins again with trapezium rule**	
	<table><tr><td>x</td><td>2</td><td>5</td><td>8</td><td>11</td></tr><tr><td>y</td><td>5.2425</td><td>2.251</td><td>1.762</td><td>1.550</td></tr></table>	M1
	Uses $\frac{1}{2}\times 3 \times \underline{\{5.2425 + 1.550 + 2(2.251 + 1.762)\}}$	M1
	$= 22.23$	A1
		[3]
	Notes	

(a) **B1**: for using $\frac{1}{2}\times 3$ or 1.5 or equivalent or just states $h = 3$

M1: requires the correct $\{\ldots\ldots\}$ bracket structure. It needs the first bracket to contain first y value plus last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed
(An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values
A1**: for the completely correct bracket** $\{\ldots\ldots\}$

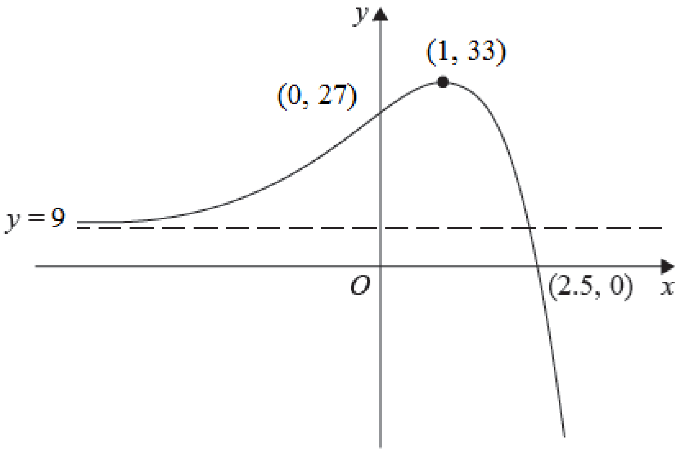
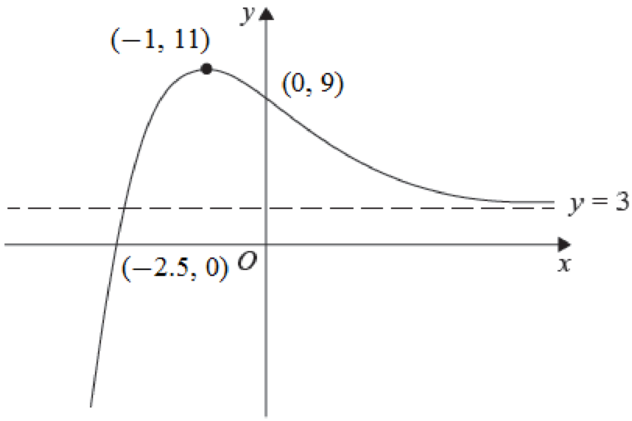
A1: for answer which rounds to 26.46 after attempt at trapezium rule
NB: Separate trapezia may be used: B1 for 1.5, M1 for 1/2 $h(a + b)$ used 2 or 3 times (and A1 if it is all correct) Then A1 for 26.46.
Special case: Bracketing mistake $1.5\times(8.485 + 1.1) + 2(2.502 + 1.524)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given).

(b) **Way 1:**

M1: Adds Area of Rectangle $= 1 \times 9$ or $\int 1\,dx = [x]_2^{11}$ to their "13.23" or to their "26.46" or to their "52.92"

M1: Half answer to part (a) seen
A1: Accept awrt 22.23

Or Way 2: (If they **begin again with a trapezium rule)**
M1: for correct table **M1:** for correct use of trapezium rule
A1: awrt 22.23

Question Number	Scheme	Marks
3.		
(a)	 Shape- similar to before but with indication of **stretch** in y direction by at least **one correct** from the three traits: y intercept, (0, 27) maximum point (1, 33) **or** asymptote indicated at 9	B1
		B1
	Intercept (0,27), max (1,33) and x intercept (2.5,0) **all three of these seen**	B1
		[3]
(b)	 Shape (reflection in y axis)	B1
	(–1,11), (0,9) and (–2.5,0) seen	B1
	$y = 3$ (must be equation)	B1
		[3]
		6 marks
	Notes	

(a)**B1**: Correct shape with curve crossing x axis and one label correct from the three listed (i.e. a correct new y value). Condone "slight" imperfections in the curvature of the sketches.
B1: All three specified labels given to indicate the three new point positions. Do not need coordinates if clearly labelled on the axes. Accept 27 and accept 2.5 and even allow (27, 0) and (0, 2.5) on y and x axes respectively.
B1: Equation of asymptote correct (asymptote on figure takes precedence) Asymptote does not need to be drawn dotted.

(b)**B1**: Correct shape (maximum in 2nd quadrant, intercept on negative x - axis and approaches asymptote for large positive x) Condone "slight" imperfections in the curvature of the sketches.
B1: All three specified labels given to indicate the three new point positions. Accept 9 and accept -2.5 and even allow (9, 0) and (0, -2.5) on y and x axes respectively.
B1: Equation of asymptote correct (asymptote on figure takes precedence) Do not award this mark if they merely copy the original graph.

If there is no sketch – the maximum mark in part (a) is B0B1B1 and in part (b) is B0B1B0 so 3/6

Special case: Stretch in y direction of scale factor 1/3. If there is a graph of the correct shape with (0,3), (1, 11/3), (2.5,0) and asymptote $y = 1$ then award B0B0B1

Question Number	Scheme	Marks
4.	(a) $\left(2+\frac{x}{4}\right)^{10} = 2^{10} + \binom{10}{1}2^9.\left(\frac{x}{4}\right) + \binom{10}{2}2^8.\left(\frac{x}{4}\right)^2 + \binom{10}{3}2^7.\left(\frac{x}{4}\right)^3 ...$	M1
	$= 1024, +1280x + 720x^2 + 240x^3 ...$	B1, A1 A1
		[4]
	(b) State or Use $x = 0.1$	B1
	Estimate = $1024 + "1280" \times 0.1 + 720 \times (0.1)^2 + 240 \times (0.1)^3 ...$	M1
	= 1159.44 or 1159.440 or 1159 or 1159.4 (after correct working)	A1
		[3]
		7 marks
	Notes	

(a) **M1**: The **method** mark is awarded for an attempt at Binomial to get one or more of the terms in x – need **correct** binomial coefficient multiplied by the correct power of x. Ignore bracket errors or errors (or omissions) in powers of 2 or 4 or bracket errors. Accept any notation for ${}^{10}C_1$, ${}^{10}C_2$ and ${}^{10}C_3$, e.g. $\binom{10}{1}$, $\binom{10}{2}$ and $\binom{10}{3}$ (unsimplified) or 10. 45 and 120 from Pascal's triangle This mark may be given if no working is shown, but any of the terms including x is correct.

B1: must be simplified to 1024 (writing just 2^{10} is **B0). If miscopied later then isw**

A1: is cao and is for **two correct** from 1280 x, $720x^2$ and $240x^3$

A1: is c.a.o and is for all of 1280 x, $720x^2$ and $240x^3$ correct (ignore extra terms) if divided by 2 or 4 then isw
Allow terms given separately without + signs and with commas. Ignore extra terms. Ignore subsequent work once correct answer is seen in simplified form.

N.B. If the series is given **in Descending Order** the first M mark may be awarded and if the whole expansion is given (all 11 terms) then full marks is possible.

(b) **B1:** States or Uses $x = 0.1$

M1: Uses their solution of $\frac{x}{4} = 0.025$ substituted in to their series expansion – If no equation stated could see evidence of use of 0.1 or
0.01 (not 0.025) substituted consistently for example

A1: This is cao and must follow M1.

NB 1159.45 or 1159.44533 is A0 (used 2.025^{10}) But correct working followed by an answer 1159 or 1159.4 can be awarded A1

Question Number	Scheme	Marks
5. (a)	$S_n = a \quad + \quad (a+d) \quad + \quad (a+2d) + ... \quad + (a+(n-1)d)$	M1
	$S_n = (a+(n-1)d) + (a+(n-2)d) \quad + ... \quad + (a+d) \quad + a$	M1
	$2S_n = (2a+(n-1)d) + (2a+(n-1)d) \quad + ... \quad + (2a+(n-1)d)$	M1
	$S_n = \frac{n}{2}[2a+(n-1)d]*$ See notes below for those who use triangle numbers in their proof	A1* **[4]**
(b)	**Uses** either $\frac{n}{2}(2\times a+(n-1)7)$ or $\frac{n}{2}(a+497)$ or $7\times\sum_{i=1}^{71} i$	M1
	i.e $\frac{71}{2}(2\times 7+70\times 7)$ or $\frac{72}{2}(2\times 0+71\times 7)$ or $\frac{71}{2}(7+497)$ or $7\times\frac{71}{2}(72)$	A1
	$= 17892$	A1 **[3]**
		7 marks
	Notes	

(a) **M1**: List terms including **at least first two** and **a last term which may be *a* + *nd* or *a* + (*n* – 1)*d* or *L***
M1: List terms in reverse including **at least their last term (or correct last term) and finally their first term**
M1: The LHS should be 2*S*. The RHS must follow from at least two terms correctly matching in the addition and should include at least two terms which are each **correctly** $\{2a+(n-1)d\}$ or $(a + L)$ **or should** be $n\{2a+(n-1)d\}$ or $n(a + L)$
A1: Need some indication of at least three terms being added (i.e at least three terms and their pairs listed with terms correctly matching or three additions seen) and also need to achieve final answer with no errors and if L was used need to state that $L = a + (n - 1)d$
NB: Some candidates use a variation of

$$\sum_{r=1}^{n}(a+(r-1)d = \sum_{r=1}^{n} a + d\sum_{r=1}^{n}(r-1) = na + d\frac{n}{2}(n+1) - dn \text{ or } na + d\frac{(n-1)}{2}(n)$$

And conclude that $S_n = \frac{n}{2}[2a+(n-1)d]$. This gains the full 4 marks M1M1M1A1, but must be completely correct.

(b) **M1:Uses** correct formula (with their a and n) with d =7 or with last term correct
A1: Uses consistent and correct a and n
A1: Correct answer

Question Number	Scheme	Marks
6.	(a) Use or state $2\log_4(2x+3)=\log_4(2x+3)^2$	M1
	Use or state $\log_4 4=1$ or $4^1=4$	M1
	Use or state $\log_4 x+\log_4(2x-1)=\log_4 x(2x-1)$ or $\log_4(2x+3)^2-\log_4 x=\log_4\frac{(2x+3)^2}{x}$ etc	M1
	$(2x+3)^2=4x(2x-1)$ or equivalent including correct rational equations	A1
	Then $4x^2+12x+9=8x^2-4x$ and so $4x^2-16x-9=0$ *	A1* **[5]**
	(b) $(2x+1)(2x-9)=0$ so $x=$ (or use other method e.g formula or completion of square)	M1
	$x=(-\frac{1}{2}$ or $)\ \frac{9}{2}$	A1 **[2]**
		7 marks
	Notes	

(a) M1**:** Uses power law for logs

M1: Connects 1 with 4 correctly

M1: Uses addition (or subtraction) law correctly

e.g. $\log_4 x+\log_4(2x-1)=\log_4 x(2x-1)$ or $\log_4(2x+3)^2-\log_4 x=\log_4\frac{(2x+3)^2}{x}$ or

$\log_4(2x+3)^2-\log_4 x-\log_4(2x-1)=\log_4\frac{(2x+3)^2}{x(2x-1)}$ or even $\log_4 x+\log_4 4=\log_4 4x$ or

$\log_4(2x-1)+\log_4 4=\log_4 4(2x-1)$ or $\log_4(2x-1)+\log_4 4+\log_4 x=\log_4 4x(2x-1)$ etc…

A1: Correct equation (unsimplified) after correct work. e.g. $\frac{(2x+3)^2}{x(2x-1)}=4$

A1: Obtains printed answer correctly (This is a given answer so needs previous A mark to have been awarded and needs correct expansion)

Special case :

$$\log_4(2x+3)^2=1+\log_4 x(2x-1)\quad so\quad \frac{\log_4(2x+3)^2}{\log_4 x(2x-1)}=1\quad so\quad \frac{4x^2+12x+9}{2x^2-x}=4$$

This can have M1, M1, M1, A0, A0 so 3/5 losing accuracy because of the error in the second step.

(b) Some candidates who did not achieve marks in part (a) begin the log work again and make more progress here. Mark the better work. So credit for (a) may be given in (b). Credit for (b) should not be given in (a)

M1: Uses solution of their quadratic or of printed quadratic(see notes). This must be in part (b)

A1: x = 4.5 and discards x = - 0.5 (any equivalent form) Giving $x=-\frac{1}{2}, \frac{9}{2}$ is A0 This must be in part (b)

Question number	Scheme	Marks
7 (a)	Obtain $(x \pm 5)^2$ **and** $(y \pm 3)^2$	M1
	Centre is $(-5, 3)$.	A1 [2]
(b)	See $(x \pm 5)^2 + (y \pm 3)^2 = 16 (= r^2)$ or $(r^2 =)"25"+"9"-18$	M1
	$r = 4$	A1 [2]
(c)	Use $x = -3$ in either form of equation of circle to obtain simplified quadratic in y	M1
	e.g $x = -3 \Rightarrow (-3+5)^2 + (y-3)^2 = 16 \Rightarrow (y-3)^2 = 12$ *or* $(-3)^2 + y^2 + 10\times(-3) - 6y + 18 = 0 \Rightarrow y^2 - 6y - 3 = 0$	
	solve resulting quadratic to give $y =$	M1
	$y = 3 \pm 2\sqrt{3}$	A1, A1 [4]
		8 marks
Alternatives (a)	*Method 2:* From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$	M1
	Centre is $(-g, -f)$ **, and so centre is** $(-5, 3)$.	A1
OR	*Method 3:* Use any value of y to give two points (L and M) on circle. x co-ordinate of mid point of LM is "-5" **and** Use any value of x to give two points (P and Q) on circle. y co-ordinate of mid point of PQ is "3" (Centre – chord theorem) . (-5, 3) is M1A1	M1 A1 (2)
(b)	*Method 2*: Using $\sqrt{g^2 + f^2 - c}$ or $(r^2 =)"25"+"9"-18$ $r = 4$	M1 A1 (2)
(c)	*Method 2*: Divide triangle PTQ and use Pythagoras with $r^2 - (-3 - "-5")^2 = h^2$, then evaluate $"3 \pm h"$ - then get $3 \pm 2\sqrt{3}$	M1 M1 A1 A1 (4)
	Notes	

Mark (a) and (b) together

(a) **M1** as in scheme and can be <u>implied</u> by $(\pm 5, \pm 3)$ **A1:** for correct centre and **(-5, 3) (without working) implies M1A1**

(b) **M1** for a complete and correct method leading to $r^2 = "25"+"9"-18$ or $r = \sqrt{"25"+"9"-18}$

or for using equation of circle in $\underline{(x \pm 5)^2} + \underline{\underline{(y \pm 3)^2}} = k^2$ form to identify $r=k$

N.B. $r^2 = k$ or $r = k^2$ is M0 Also $-$ "25" $-$ "9" $-$ 18 is M0 and $r^2 = "25"+"9"$ (without the 18)is M0

A1 r **=** 4 (only and not with r = -4) Again **correct answer with no working implies M1A1**

Special case: if centre is given as $(5, -3)$ **or (5, 3) or (-5, -3)** allow **M1A1** for r = 4 worked correctly as $(r^2 =)"25"+"9"-18$ i.e if they obtain r =4 after sign error give final A1 (So M1A0M1A1)

(c) M1 For substituting x = -3 into an equation for the circle and attempt to simplify to 3 term quadratic or to $(y-a)^2 = b$

M1 For attempting to solve their quadratic (following usual rules – see notes)

A1, A1 Answers must be given as surds – A1 for each correct answer. To earn both A marks, answers must be simplified.

Question Number	Scheme	Marks
8.		
(a)	$u_2 = 3k - 12,\ u_3 = 3(u_2) - 12$	M1
	$u_2 = 3k - 12,\ u_3 = 9k - 48$	A1
	$u_4 = 3(9k - 48) - 12 = 27k - 156$ (ft their u_3) .	M1 A1ft
		[4]
(b)	$27k - 156 = 15$ so $k =$	M1
	$k =\ 6\frac{1}{3}$ or $\frac{19}{3}$ or 6.33 (3sf)	A1
		[2]
(c)	$\sum_{i=1}^{4} u_i = 6\frac{1}{3} + 7 + 9 + 15$ or $\sum_{i=1}^{4} u_i = k + 3k - 12 + 9k - 48 + 27k - 156$	M1
	$= 40k - 216$, $= 37\frac{1}{3}$ or $\frac{112}{3}$	A1ft, A1cao
		[3]
		9 marks
	Notes	

(a) **M1:** Attempt to use formula twice to find u_2 and u_3

A1: two correct simplified answers

M1: Attempt again to find u_4

A1ft: 4th term correct and simplified - follow through their u_3

(b) **M1**: Put their 4th term (not 5th) equal to 15 and attempt to find $k =$

A1: accept any correct fraction or decimal answer (allow 6.33 or better here)

(c) **M1:** Uses 1st term and their following 3 terms **with plus signs** (either numerical or in terms of *k*). Must be using terms from iteration and not formula for an AP or GP. May make a copying slip.

A1ft: for $40k - 216$ or follow through on their *k* so check $40k - 216$ for their *k*

A1: obtains $37\frac{1}{3}$ (must be exact) if exact answer given, then isw

Those who use 6.3 will obtain 36 They should have M1A1ftA0 – should have used exact *k* to give exact answer here.

Those who use 6.33 will obtain 37.2 This should have M1A1ftA0 – should have used exact *k* to give exact answer here.

Those who use 6.333 will obtain 37.32 This should have M1A1ftA0 – should have used exact *k* to give exact answer here.

6.3333 will obtain 37.332 This should have M1A1ftA0 – should have used exact *k* to give exact answer here.

6.33333 will obtain 37.3332 etc All these answers should have M1A1ftA0 – should have used exact *k* to give exact answer here. Etc

Special case: Those who use $k = 6$ will obtain $6 + 6 + 6 + 6 = 24$ This is M1 A0 A0 in part (c) – as over simplified

Question Number	Scheme	Marks
9. (a)	$5^2 = 10^2 + 12^2 - 2\times 10\times 12\cos\angle XAB$, or $\cos\angle XAB = \frac{10^2+12^2-5^2}{2\times 10\times 12}$ or $\frac{219}{240}$ or 0.9125 or $\frac{73}{80}$	M1
	$\angle XAB = 0.421$ or 0.134π	A1 **[2]**
(b)	Area of sector is $\frac{1}{2}r^2\theta = \frac{1}{2}\times 10^2\times\theta$	M1
	Area of major sector is $\frac{1}{2}\times r^2(2\pi - 2\times"0.421")$ or $\pi\times r^2 - \frac{1}{2}\times r^2\times 2\times"0.421")$	M1
	$= 272$	A1 **[3]**
(c)	area of triangle $AXB = \frac{1}{2}10\times 12\times\sin XAB$ — **Way 2**: Find angle XBA and hence area XBY	M1
	area of kite $= 2\times$ triangleAXB — Area of kite = area of XBY + Area XAY	dM1
	= awrt 49 — = 37.298 + 11.76 = 49	A1 [3]
	Way 3: Finds length XY by cosine rule or elementary trigonometry (8.173)	
	Uses area of kite = $\frac{1}{2}"8.173"\times 12$	M1
	= awrt 49	dM1
		A1 [3]
		8 marks
	Notes	

(a) **M1**: Uses cosine rule – must be a correct statement , allow statement $5^2 = 10^2 + 12^2 - 2\times 10\times 12\cos\angle XAB$
A1: accept awrt 0.421 (answers in degrees gain M1 A0). Also 0.42 is A0

(b) **M1**: Uses area formula with r = 10 and any angle in radians. If they use degrees they must use the formula $\frac{\theta}{360}\times\pi 10^2$
M1: Finds angle in major sector ft their angle from (a) and uses sector formula **or** subtracts minor area from circle (allow work in degrees) Must use $(2\pi - 2\times"0.421")$ but r may be 5 instead of 10 for this mark
A1: Accept awrt 272 (may reach this using degrees)

(c) Way 1: **M1**: Finds area of triangle *AXB*, using 10, 12 and their angle *XAB*
dM1: Doubles area of triangle *AXB*
Way 2: **M1**: Finds angle *XBA* (0.958..) by valid method (cosine rule) (**NOT 90 – *XAB***) and hence area *XBY* $= \frac{1}{2}5\times 5\times\sin 1.9163$
dM1: Adds areas of triangles *XBY* and *XAY* (37.298 and 11.76)
Way 3: **M1**: Finds length XY by cosine rule or elementary trigonometry (8.173)
dM1: Uses area of kite = $\frac{1}{2}"8.173"\times 12$

For each method A1: awrt 49- do not need units

Question Number	Scheme	Marks
10.	$\text{f}(x) = 6x^3 + ax^2 + bx - 5$	
(a)	Attempts $\text{f}(\pm 1)$ **or** Attempts $\text{f}(\pm\frac{1}{2})$ Or Use long division as far as remainder*	M1
	Obtains $6(-1)^3 + a(-1)^2 + b(-1) - 5 = 0$ or $-6 + a - b - 5 = 0$ or $a - b = 11$ or equivalent	A1
	Obtains $6(\frac{1}{2})^3 + a(\frac{1}{2})^2 + b(\frac{1}{2}) - 5 = -15$ or $\frac{6}{8} + \frac{a}{4} + \frac{b}{2} - 5 = -15$ or $a + 2b = -43$ or equivalent	A1
	Solve simultaneous equations to obtain $a = -7$ and $b = -18$	M1 A1
		[5]
(b)	$6x^3 + ax^2 + bx - 5 = (x+1)(6x^2 + ...x + ...)$	M1
	$6x^3 - 7x^2 - 18x - 5 = (x+1)(6x^2 - 13x - 5)$	A1
	$(6x^2 - 13x - 5) = (ax+b)(cx+d)$ where $ac = "6"$ and $bd = "\pm 5"$	M1
	$= (x+1)(2x-5)(3x+1)$	A1
		[4]
		9 marks
	Notes	

(a) **M1**: Using remainder theorem: As on scheme. One of these is sufficient do not need to equate to 0 and to -15

*Using Long division: need at least $6x^2+(a-6)x+....$ as quotient, and get as far as remainder **or for the other division reaches** $3x^2+(\frac{a+3}{2})x+....$ as quotient, and get as far as remainder .

A1: Any equivalent form *e.g. -11 – *b* + *a* = 0 (using remainder after division) The mark is earned for *a* – *b* = 11 even if "=0" not explicitly seen

A1: Any equivalent form *e.g. $-5+\frac{b}{2}+\frac{a+3}{4}=-15$ (using remainder after division) Must be accurate but may be unsimplified. NB Using 15 instead of -15 is A0

M1: Solves their **linear** equations to obtain *a* or *b*

A1: Both *a* and *b* correct. Correct answers without working can earn M1A1.

(b) M1: Recognises (*x*+1) is factor and obtains quadratic expression with correct first term by any method. Use of (*x* -1) is M0. NB Starting with (*x* + 1) (2*x* – 1) *(ax + b)* is also M0

A1: Correct quadratic $(6x^2-13x-5)$

M1: Attempt to factorise quadratic where $ac=\text{"}6\text{"}$ and $bd=\text{"}\pm 5\text{"}$

A1: any correct combination e.g. $=2(x+1)(x-\frac{5}{2})(3x+1)$ or $=6(x+1)(x-\frac{5}{2})(x+\frac{1}{3})$ etc… (on one line)

Following a correct value for *a* and for *b:*
They may just write the factorised answer down.
For a correct answer this is M1A1M1A1
For $=(x+1)(x-2.5)(x+\frac{1}{3})$ award M1A0M1A0
For **correct answer** following **incorrect quadratic** give M1 A0 M1 A0 – fortuitous
If the correct answer follows incorrect *a* and *b*, it is fortuitous and again M1A0M1A0 should be given.

Question Number	Scheme	Marks
11 (a)	$\left(0, -\frac{\sqrt{3}}{2}\right)$	B1
	and (60°, 0) and (240°, 0) and (-120°, 0) and (-300°, 0)	B1 B1 **[3]**
(b)	$\sin(x-60^\circ) = \frac{\sqrt{6}-\sqrt{2}}{4}$ **(**= 0.2588)	M1
	x – 60° = 15° (or 165° or – 195° or – 345°) or 0.262 or $\frac{\pi}{12}$ radians	A1
	So x = 75° or 225° or – 135° or –285° (allow awrt)	M1 A1 A1 **[5]**
		8 marks
	Notes	

(a) **B1** : Correct exact y intercept (not decimal) – allow on the diagram or in the text. Allow $y = -\frac{\sqrt{3}}{2}$

B1 for 2 correct x intercepts then **third B1** for all 4 correct x intercepts (may or may not be given as coordinates – may be given on graph) Must be in degrees. (Extra answers in the range lose the **third B1**)

(b) **M1**: Divides by 4 first giving correct statement $\sin(x-60^\circ) = \frac{\sqrt{6}-\sqrt{2}}{4}$ but $(x-60^\circ) = \frac{\sqrt{6}-\sqrt{2}}{4}$ **is M0 and** $\sin x - \sin 60^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$ **is also M0 and** $\sin(x-60^\circ) = \frac{\sqrt{4}}{4}$ **is M0** if not preceded by correct statement

A1: Obtains 15° (or 165° or – 195° or – 345°)

M1: Adds 60° to their previous answer which should have been in degrees and obtained by using inverse sine

A1: Two correct answers second **A1:** All four correct answers Extra answers outside range are ignored. Lose final A mark for extra wrong answers in the range.

If they approximate too early allow awrt answers given for full marks. (e.g. 75.01 etc)

Answers in mixture, degrees and radians: Allow first M A1 only so M1A1M0A0A0 for 60.262 for example

Question Number	Scheme	Marks
12.(a)	Uses $275000\times(1.1)^5$ or finds £442890.25 or uses $275000\times(1.1)^4$ or finds £402627.50	M1
	Finds both of the above and subtracts to give £40 262.75 and concludes approx. £40300*	M1 A1*
	Or	**[3]**
	Uses $275000\times(1.1)^5 - 275000\times(1.1)^4$, $= awrt\,40260 = 40300\ (3sf)$ *	M1 M1,A1*
		[3]
(b)	Puts $275000\times(1.1)^{n-1} > 1000000$ or $275000\times(1.1)^{n-1} = 1000000$	M1
	$(1.1)^{n-1} > \dfrac{1000000}{275000}$ (or $\frac{40}{11}$ or 3.63 or 3.64). Or	M1
	$(1.1)^{n-1} = \dfrac{1000000}{275000}$ (or $\frac{40}{11}$ or 3.63 or 3.64)	
	$n-1 > \dfrac{\log\left(\frac{40}{11}\right)}{\log 1.1}$ or $n-1 = \dfrac{\log\left(\frac{40}{11}\right)}{\log 1.1}$	M1
	($n > 14.5$ or $n > 14.6$ or $n = 15$) so the year is 2030	A1
		[4]
(c)	Uses $S = \dfrac{275000(1.1^n - 1)}{1.1-1}$ or uses $S = \dfrac{275000(1-1.1^n)}{1-1.1}$	M1
	Uses n = 11 in formula	A1
	Awrt £5 096 100	A1
	Or: adds 11 terms £275000 + 302500 + 332750 + 366025 + 402627.5 + 442890.25 + 487179.275 + 535897.2025 + 589486.9228 + 648435.615 + 713279.1765 = awrt 5096100 (see notes below)	**[3]**
		10 marks
	Notes	

(a) **M1**: for correct expression for profit in 2021 **or** in 2020, by any method (including subtracting the sums $S_{n+1} - S_n$) to give a term

M1: for finding both correct expressions and subtracting

A1: answers wrt£442900 and wrt£402600 subtracted **or** wrt£40260 obtained **then rounded to** £40300 (answer given)

(b) **M1**: Correct inequality – or allow equality . N.B. $250000\times(1.1)^n$ or $302500\times(1.1)^{n-2}$ on LHS are also correct.

M1: Division – isw if initial fraction is correct. Not dependent on previous mark. It could follow wrong combination of *a* and *n* for example, which would give M0 M1

M1: Correct use of logs to give *n* or $n-1>\frac{\log(k)}{\log 1.1}$ or $\log_{1.1} k$ after $(1.1)^{n-1}>k$ Allow equality for this mark

(3.63 is truncated value of $\frac{40}{11}$ and 3.64 is rounded value – allow either of these if used in place of fraction)

A1: 2030 is required . If inequalities are used and errors are seen, then this mark is A0 (even for 2030)

(Trial and improvement or listing can have full marks for the correct answer, need to see both 14th and 15th term – otherwise zero)

Special case: If *n* is used instead of $n-1$ and they reach 2029 then mark profile is likely to be M0 M1 M1 A0 **unless they recover to the correct answer when full marks may be earned**

If an equals sign is used throughout and then correct answer is obtained allow 4/4

Special case: Uses Sum formula – Can earn M0 M0 M1 A1 for "correct work"

Uses $S=\frac{275000(1.1^n-1)}{1.1-1}>1000000$ (M0) $1.1^n>1+\frac{1000000}{2750000}$ (M0) $n>\frac{\log(15/11)}{\log 1.1}$ (M1) $n>3.254....$ so 2019 (A1)

Using this method with errors can earn M0M0M1A0 for proceeding from $1.1^n>k$ with $k>0$ to $n>\frac{\log(k)}{\log 1.1}$

(c) **M1**: Correct *a* and *r* but *n* may be wrong

A1: Correct use of formula with $n = 11$

A1: awrt £5 096 100 (again – this answer implies all 3 marks)

Or M1: adds 11 terms (mostly correct)

A1: lists 11 correct terms £275000 + £302500 + £332750 + £366025 + £402627.5 + £442890.25 + £487179.275 + £535897.2025 + £589486.9228 + £648435.615 + £713279.1765

A1: correct answer = awrt £5096100 (this implies two previous marks)

Question Number	Scheme	Marks
13.	$y = 3x^2 - 4x + 2$	
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x - 4 + \{0\}$	M1A1
	At (1, 1) gradient of curve is 2 and so gradient of normal is $-\frac{1}{2}$	M1
	$\therefore (y-1) = -\frac{1}{2}(x-1)$ and so $x + 2y - 3 = 0$ *	M1 A1*
		[5]
(b)	Eliminate x or y to give $2(3x^2 - 4x + 2) + x - 3 = 0$ or $y = 3(3-2y)^2 - 4(3-2y) + 2$	M1
	Solve three term quadratic e.g $6x^2 - 7x + 1 = 0$ or $12y^2 - 29y + 17 = 0$ to give $x =$ or $y =$	M1
	$x = \frac{1}{6}$ **or** $y = 1\frac{5}{12}$	A1
	Both $x = \frac{1}{6}$ and $y = 1\frac{5}{12}$ i.e. $(\frac{1}{6}, 1\frac{5}{12})$ *or* (0.17,1.42) { Ignore (1 , 1) listed as well }	A1
		[4]
(c)	When this line meets the curve $2(3x^2 - 4x + 2) + kx - 3 = 0$	M1
	So $6x^2 + (k-8)x + 1 = 0$	dM1
	Uses condition for equal roots "$b^2 = 4ac$" on their three term quadratic to get expression in k	ddM1
	So obtain $(k-8)^2 = 24$ i.e. $k^2 - 16k + 40 = 0$ *	A1 *
	If they use **gradient of tangent** to do part (c) see the end of the notes below*.	**[4]**
(d)	Solve the given quadratic or their quadratic by formula or completion of the square to give	M1A1
	$k = 8 \pm \sqrt{24}$ or $8 \pm 2\sqrt{6}$ or $\frac{16 \pm \sqrt{96}}{2}$	**[2]**
		15 marks
	Notes	

(a) **M1:** Evidence of differentiation, so $x^n \to x^{n-1}$ at least once
A1: Both terms correct
M1: Substitutes x = 1 into their derivative and uses perpendicular property
M1: Correct method for Linear equation, using (1,1) and their changed gradient
A1: Should conclude with printed answer (this answer is given in the question)
(b) M1: May make sign slips in their algebra; {e.g. substitute 3 + 2y }– does not need to be simplified so isw.
But putting $3(3-2y)^2 - 4(3-2y) + 2 = 0$ instead of $= y$ is M0
M1: Solve three term quadratic to give one of the two variables
A1: One Correct coordinate – accept any equivalent
A1: Both correct – any equivalent form. Allow decimals if correct awrt (0.17, 1.42) (ignore (1,1) given as well)
(c) M1: Eliminate y (condone small copying errors)
dM1: Collect into 3 term quadratic in x or identifies "a", " b" and " c" clearly (may be implied by later work).
ddM1: Uses condition $"b^2 = 4ac"$ on quadratic in x (dependent on both previous M marks)
NB **M0** for $b^2 > 4ac$ or $b^2 \geq 4ac$ or $b^2 < 4ac$ or $b^2 \leq 4ac$
A1: Need $(k-8)^2 = 24$ or equivalent before stating printed answer
*Alternative method for part (c)
M1: Use gradient of line = gradient of curve so $"6x-4" = "-\frac{k}{2}"$
M1: Find $x = \frac{2}{3} - \frac{k}{12}$ and use line equation to get $y =$ $\frac{3}{2} - \frac{1}{3}k + \frac{k^2}{24}$ (these equations do not need to be simplified)
M1: Find $x = \frac{2}{3} - \frac{k}{12}$ and use curve equation to get $y =$ $\frac{2}{3} + \frac{k^2}{48}$ (these equations do not need to be simplified)
A1: Puts two correct expressions for y equal and obtains printed answer without error.
(d) M1: Solve by formula or completion of the square to give $k =$ (Attempt at factorization is M0)
A1: Correct answer – should be one of the forms given in the main scheme or equivalent exact form
Answers only with no working 2 marks (exact and correct) or 0 marks (approximate or wrong)

Question Number	Scheme	Marks
14. (i)	Way 1: Use $\frac{\sin x}{\cos x} = \tan x$ to give $\tan x =$ Way2: complete method to find sinx = or cosx =	M1
	$\tan x = -\frac{7}{3}$ or $\sin x = \pm\frac{7}{\sqrt{58}}$ or $\cos x = \pm\frac{3}{\sqrt{58}}$	A1
	So x = 113.2, 293.2	M1 A1 **[4]**
(ii)	$10\cos^2\theta + \cos\theta = 11(1-\cos^2\theta) - 9$	M1
	Solves their three term quadratic "$21\cos^2\theta + \cos\theta - 2 = 0$" to give $\cos\theta = ...$	M1
	So $(\cos\theta =)\ -\frac{1}{3}$ or $\frac{2}{7}$	A1
	$\theta = 1.91,\ 4.37,\ 1.28$ or 5.00 (allow 5 instead of 5.00)	M1 A1 A1 **[6]**
		10 marks
	Notes	

(i) M1: (Way 1) Attempts to use $\frac{\sin x}{\cos x} = \tan x$ (there may be a sign error or may omit x and write tan =)

(Way 2) $3\sin x = -7\cos x$ so $9\sin^2 x = 49\cos^2 x$ and uses $\sin^2 x + \cos^2 x = 1$ to find sin$x$ = or cos$x$ =

A1: must be $\tan x = -\frac{7}{3}$ (way 1) or allow $\sin x = \pm\frac{7}{\sqrt{58}}$ or $\cos x = \pm\frac{3}{\sqrt{58}}$ (way 2). Ignore cos$x$ = 0 as extra answer.

M1: One correct angle in degrees in range – so need either 113.2 or 293.2 in most cases

But If they had $\tan x = -\frac{3}{7}$, then obtaining 156.8 or 336.8 is equivalent work and gains M1

If however they had $\tan x = +\frac{7}{3}$, then obtaining an answer in the range is not equivalent work – so is M0

A1: These two answers - accept awrt 113.2 and 293.2 Extra answers in range – lose this mark
Working in radians gives a maximum of M1A1M0A0

(ii) M1: Replaces $\sin^2\theta$ by $(1-\cos^2\theta)$

M1: Collects terms and solves their three term quadratic by usual methods (see notes)

A1: Both correct answers needed , but isw if one then rejected. Allow awrt -0.333 and 0.286

M1 Uses inverse cosine to obtain at least two correct answers for their values of cosine (check with calculator if they have followed wrong values)

A1: Any two completely correct answers (allow awrt)

A1: All four correct (awrt) Allow 0.608π, 1.39 π, 0.408 π, or 1.59 π

Extra answers outside range – ignore **Extra answers in the range** – lose final mark. Inaccurate answers to 3sf lose final A mark

Answers in degrees lose final two marks

So two of awrt 73, 287, 109 (or 109.5), 251 (or 250.5) would earn M1A0A0

Question Number	Scheme	Marks
15.	$y = x^3 + 10x^{\frac{3}{2}} + kx$	
(a)	$\frac{dy}{dx} = 3x^2 + 10 \times \frac{3}{2}x^{\frac{1}{2}} + k$	M1 A1 [2]
(b)	Substitutes $x = 4$ and $\frac{dy}{dx} = 0$ to give $3(4)^2 + 15(4)^{\frac{1}{2}} + k = 0 \quad \Rightarrow k = -78$ *	M1 A1* **[2]**
(c)	When $x = 4$, $y = -168$ (see this stated – or see rectangle has height 168)	B1
	$\int x^3 + 10x^{\frac{3}{2}} - 78x\ (+168)\,dx = \frac{1}{4}x^4 + \frac{10}{\frac{5}{2}}x^{\frac{5}{2}} - \frac{78}{2}x^2 (+168x + c)$	M1 A1
	Use limits 0 and 4 to give ±432 or if 168x included to give ±240	dB1
	Rectangle area is 4 × "168" (= 672) or see 168x in integrated answer with limits	M1
	So R has area "672 – 432" or see +168 in original integrand	M1
	= 240	A1 **[7]**
		11 marks
	Notes	
(a)	**M1:** Fractional power dealt with correctly so becomes $\frac{3}{2}x^{\frac{1}{2}}$ (may be implied by simplification to 15) **A1:** All terms correct, may not be simplified	
(b)	**M1: Substitutes** $x = 4$ and $\frac{dy}{dx} = 0$ Must see $3(4)^2 + 15(4)^{\frac{1}{2}} + k = 0$ or $48 + 30 + k = 0$ ***A1:** This is a printed answer so all must be correct in the working and conclusion k = -78 is needed.	
(c)	**B1:** Substitute into y = to find y (This may appear anywhere in the answer) **M1:** Attempt to integrate so at least one power increases **A1:** Accept unsimplified correct answer and allow with or without their +168x, or even with their -168x **dB1**: Use limit 4 to give 432 but may be implied by later answer 240- needs to follow M1A1 for integration **M1:** Calculates rectangle area (may be by integration). Must be rectangle and not triangle area **M1:** Subtracts (either way round) numerical areas – should be (+) – (+) or (-) - (-) (subtraction may be in their original integral but penalize wrong sign here eg -168x instead of +168x) (Again use of triangle is M0) **A1:** 240 only (Can recover from -240 to 240) Common error: If 168x (instead of 168) is integrated this may only gain a maximum of B1 M1 A1 dB1 (for seeing 432 calculated if integrals are separated) M0 M0 A0 4/7	